Recent Usage of Software Tools in the Teaching and Learning of Engineering Mathematics

N. Lohgheswary

Z. M. Nopiah

E. Zakaria

Abstract

This paper investigates the recent usage of software tools in the teaching and learning of engineering mathematics. The scope of this paper is on the software's that are mostly used in the teaching and learning of Vector Calculus, Linear Algebra and Differential Equation. As many as 21 papers were selected based on the usage of MATLAB, Maple or Mathematica in the teaching and learning of engineering mathematics. The study indicates that types of software used depend on the subject learning.

Keywords: *Component; engineering mathematics; Vector Calculus, Linear Algebra, Ordinary Differential Equation; Maple; MATLAB; Mathematica; teaching and learning*

I. Introduction

Engineering is a branch of science and technology that combines knowledge, mathematics, and experience to design an object or process. Most of the engineering problems involve mathematical equations and manipulation which require every engineering practitioner to master the mathematics knowledge and comprehension. There are many approaches used to enhance the process of teaching and learning in engineering mathematics, including the usage of computer and software. The use of mathematical software such as MATLAB, Maple and Mathematica can stimulate students' interest in mathematical problems while reinforcing knowledge and understanding,

Faculty of Engineering and Built Environment, [2]Centre for Engineering Education Research, Universiti Kebangsaan Malaysia, Bangi 43600, Malaysia. Email: zambri@ukm.edu.my

Centre for Engineering Education Research, Faculty of Engineering & Built Environment, Universiti Kebangsaan Malaysia, 43600 UKM Bangi, Selangor, Malaysia

Faculty of Education, Universiti Kebangsaan Malaysia, 43600 UKM Bangi, Selangor, Malaysia. Email: zmn@ukm.edu.my, effandi@ukm.edu.my

especially for engineering students. This paper reviews the development on the usage of the software in the teaching of engineering mathematics.

II. Vector Calculus

One of the fundamental subjects in every engineering course is calculus vector. It is an extension of calculus with the combination of vector properties. It is one of the most difficult subjects in the engineering mathematics at the early years of engineering programme.

Tonkes et al. [1] conducted a study on the usage of MATLAB at the University of Queensland. In pre-2002, labs were conducted forth nightly at the University of Queensland where students underwent a self-study where they were provided with worksheets containing syntaxes and problems. In 2002 MATLAB labs were conducted weekly together with exercises and solutions. From 2003 onwards a learning model was launched in the University of Queensland for the first year syllabus using MATLAB. Eight hundred first year students participated in this project with a total of 50 lab classes. The learning model was revised for a three-year time span. Courses in MATLAB include Precalculus, Calculus of one variable, Calculus of many variables, Linear Algebra and Ordinary Differential Equations. Students used a structured workbook which consists of several modules. Every module targets at one important concept. The module consists of mathematical concepts, examples and exercises based on MATLAB that was tested on a fifty-five minute duration. First of all, the students ran simple commands to familiarize and to gain confidence on using MATLAB. Later they used GUIs from MATLAB to visualize advanced mathematical concepts in a computational manner. Finally students developed their own coding before they ran the prewritten programs. Student survey reported that students were positive about incorporating software in the mathematics curriculum. GUI experiments successfully visualize difficult concepts. Although students worked in pairs during the implementation of the learning model, they preferred to work individually. The learning model has improved the learning outcomes over the years. However the software packages faced some resistance from students due to three main reasons. Firstly, the software MATLAB, was rather numerical than symbolic. Secondly students could not see the link between laboratory exercises and course material. Lastly it was difficult to write and execute the program.

Cook [2] used Maple graphing tool in teaching three-dimensional Calculus. It was very challenging even though lecturers spent ample time on their illustrations to get the students to understand the underlying concept. Yet students still faced some difficulties to grasp the theory. Visualization technique was used for understanding three dimensional Calculus. To overcome this problem, Cook [2] wrote examples of eight functions in calculus III plots using Maple. These examples were extended from classroom lessons to class projects. The examples include plotting Riemann Sums for the functions of two variables, Lagrange multipliers, line integrals and plotting secant vectors. The aim of these examples was not to make the students proficient in computational mathematics but rather to provide them with a better understanding on what was being computed. Synder [3] studied the instrumentation process of students' learning using the command-driven Maple during a semester Calculus course. Assignments were aimed to improve the depth of students' conceptual understanding of Calculus and usage of Maple as a problem-solving tool. A collective case study was implemented to examine students' proficiency in using the computer algebra system. Before distributing the assignments, two class sessions were facilitated in the computer lab at the beginning of the term to assist students in learning Maple. Individual tutorials assisted students in learning the commands and techniques of Maple. Maple was installed in all the computer labs on campus. Students were given one group assignment and five individual assignments every two weeks in order to help them learn Maple. The duration for each assignment was one week. All activities such as learning the software, interviews and completing assignments were videotaped and analyzed. Since the tutorial exercises were directed at the usage of each command, students barely had the chance to experience it themselves. Besides the fact that students did not spend most of their time to experiment this software they also did not have access to it in their laptops. To realize the benefit of the computer algebra system like Maple, Synder[3] suggested that it must be integrated into the course where Maple should be frequently used in class, regular homework assignments integrated with Maple and certain assessment from the software. Assessment includes verifying answers, correcting errors and analyzing results.

To improve the effectiveness of the study process, Kovacheva[4] applied two components of educational technologies. Technologies used in the study process include the computer algebra system. Technologies of the study process include planning, organizing, carrying out and evaluating the entire process. Maple was incorporated in Calculus, Linear Algebra, Ordinary Differential Equations, Numerical Methods and Statistics. Each subject consists of seven laboratory exercises. Although the work method developed students' skills for team work, it is thought that many students with one computer was a condition not deemed not ideal. Laboratory exercises using the Maple system was aimed to extend students' knowledge acquired in lectures and expose them to work with a ready-made system. Students were exposed to numerical methods of problem-solving using Maple. Laboratory exercises enhanced the depth comprehension of a subject and increased students' motivation.

Suanmali [5] employed Maple as a multimedia tool in classrooms. Students who enrolled in Calculus courses for engineering majors were required to learn and use Maple in their assignments. Maple's user friendly interface and the help sessions developed students' proficiency to operate commands and codes. The complex theory and concepts of Calculus can be solved by visualizing the problems. Traditionally, it was difficult for students to learn Riemann Sum *for* $y = f(x)$ on the interval $[a,b]$. Firstly, they need to sketch the region. Then by finding the area of inscribed and circumscribed rectangles they can approximate the area under the curve. Only then they will be exposed to the concept of definite integral. By generating figures to plot graphs and calculate approximate area using various numbers of rectangles using Maple, students were able to visualize the insight of the Riemann Sum concept. These multimedia tools excited the students and also prevented them from losing sight of the subjects. Maple turned to be significant in students' learning as it improves lecturers and students' problem solving skills via technology.

Dikovic' [6] explored the effects of using GeoGebra in Differential Calculus. The lectures and exercises were conducted in the traditional method. Later the experimental group which consisted of 31 students worked in a computer laboratory. The students were freely allowed to have group discussion. The teacher became a coordinator. Help was provided by the teacher on the student-to-student basis. The traditional teaching method was substituted by group work, individual research and investigations. Before the session began, a pre-test consisted of ten simple tasks was conducted and at the end of the session a post-test was carried out. The average score for the pre-test was M = 22.95 and for the post-test it was M = 51.64. The results proved that GeoGebra is a powerful tool for the visualization and simulation of important topics of Differential Calculus. Furthermore the computing tool impacted positively on the understanding and knowledge of Differential Calculus among the experimental group members.

Godarzi et al. [7] investigated the development of procedural and conceptual knowledge in the teaching-learning double integral via Maple 12. A total of forty-four students from the second semester from Rajee Teacher Training University were chosen randomly for this study. They were divided into the experimental group and control group. Control group learnt using the traditional way while the experimental group used Maple 12 as the mode of study. There were six sessions with one and half hours allocated for both groups. A pre-test and post-test were administered for both groups. The pre-test was aiming to identify students' previously acquired knowledge. There were no significant differences on the students' pre-test score for both groups. The post-test however showed that the experimental group had a better conceptual and procedural understanding than the control group. Ten students were interviewed from the experimental group to analyze students' depth of knowledge and to gather their opinion related to Maple 12 software. Most students agreed that Maple 12 was helpful in visualizing the basic concepts of Multivariable Calculus. Students demonstrated higher level of confidence, positive attitude towards mathematics and performed better result.

Noinang et al. [8] created an efficient Mathematics teaching-learning tool for Integral Calculus courses which included PowerPoint slides with Maple animation and interactive Maplets with Maple worksheets. The teaching-learning tool aided the instructors to conduct lectures. It also helped the students in self-planned learning and self-assessment. Line integrals, surface integrals and volume integrals were taught using Maplets. The fundamental step by step concepts were learnt through Maplets. Each Maplet covers three main functions which include some input functions to define a problem with help sections, a graphic visualization function and output functions for demonstrating the results. Maplets, which supports Mathematics concepts graphically, leads the students to achieve higher level of logical analytical thinking. In order to measure students' achievements, pre and post tests were conducted. Both pre and post-tests consist of basic concepts and application-based questions. Basic concepts were tested using multiple choice tests and short answer tests. Application questions were in the form of writing test. Maplets with Maple worksheets were used to check students' answers for homework. With a quick solution and enhanced visualization interactive Maplets and Maple worksheets, it reinforced students' conceptual understanding of Integral Calculus.

Botana et al. [9] used Sage, a free open source software, in the teaching of Advanced Calculus. A live DVD of Sage was developed which contains thirty Sage interacting as the complementary material for first year students from the School of Forestry Engineering from the University of Vigo, Spain. Students were encouraged to bring along their laptops in classes. Half of the students brought laptops while the other half shared with their friends. Sharing students were found to learn the interface faster. Students were assigned tasks and they needed to solve some problems using interacts and returned as Sage worksheets. Students commented that these interacts avoided time-wasting in uninteresting mechanical computations. They preferred to concentrate in concepts instead.

III. Linear Algebra

According to Charney [10], at Virgina Tech, the course Matrix Methods of Structural Analysis seemed to be challenging as many of the students had little exposure to programming. This course needs the basics of Linear Algebra and Theory of Structures. To minimize the challenges and to provide a firm theoretical basis in the matrix structural analysis, the course namely Computer Methods of Structural Analysis I was implemented. Since Mathcad is commonly used in the structural engineering profession, it was incorporated in the course. Additionally, Mathcad is easy to learn and highly visual. Mathcad was used as a visual matrix manipulation tool and for writing complete structural analysis programs. The computer-based structural analysis subject requires student to understand well the flow of the program and it can only be achieved if the student experiences the implementation of the theory. Complex structure, for instance, cantilever beam which is hardly ever analyzed by the written calculation could be easily done by integrating Mathcad.

Kilicman et al. [11] used Maple in nine interactive tutorials in Linear Algebra. This includes plotting the eigenvector, computing the eigenvalues and solving the linear system. Subjects such as Real Analysis and Applied Mathematical Courses consist of applications of eigenvalues and eigenvectors. These difficult topics become easy with the aid of Maple. Incorporating technologies such as Maple provides a hand-written solution to help students to correct and evaluate their work. ICT tools help students to learn material in-depth and in a resourceful way. Students' skills and ability were improved and they were able to connect concepts during the problem-solving process. Maple aids both theoretical and computational aspects of the Linear Algebra courses.

The advanced computer algebra system program, Maple was also used in the first year engineering mathematics course particularly Linear Algebra at The Technical University of Denmark. Schmidt et al. [12] discussed the challenges and benefit that can come from the implementation of the computer algebra system. Maple was considered an important element in teaching and also viewed as a tool for visualization at lectures. Group exercises as well as compulsory projects were assigned in Maple. During the period of 2007 to 2008, surveys were conducted to find the relationship between study resources and study activities. Textbooks, the Maple Demos, the Internet and course materials were classified as study resources while lectures, group exercises, quizzes and compulsory homework were categorized as study activities. Students' feedback was recorded in week three, nine and nineteen during the academic year which included 26 weeks of two semesters. Students participated in group exercises and the total time spent for doing homework exercises was relatively very high. However, there were some challenges that arise in implementing Maple in the Engineering Mathematics curriculum. Both students' attendance for the lecture and students' preparation for classes were on the decline. Maple Demos overtook textbook as study resources by achieving higher results.

Van et al. [13] developed intelligent educational software and used it to implement a system at automatic problem solving in Linear Algebra. The system could solve problems within three domains namely Linear Equations System – Matrix – Determinant, Vector Space and Linear Map. The solutions provided were equal to human's thinking and writing. The algorithms have been implemented in Maple and the solutions through C. The systems were tested by students from the University of Information Technology. Positive feedback was received from the students where they viewed the system to be highly useful in learning Linear Algebra.

Chen [14] stated that since Linear Algebra's subject content was highly abstract with a theoretical feature, many students found that it is difficult to grasp the knowledge and application. More time was wasted in solving complex solutions by hand calculations which ended up with students getting bored with the subject. Students were introduced to the computing software particularly MATLAB to convert the mathematical theory into

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numerical computations. By using the tool, the students managed to obtain the results immediately and accurately. This stimulated their interest in studying Linear Algebra. MATLAB was also found to help students to visualize abstract concepts. Thus, this tool strengthened the theory and application and improved their ability in mathematical modeling.

IV. Ordinary Differential Equation

To enhance the learning effectiveness of numerical methods in the Master Course of Polymers Engineering at the University of Minho, Carneiro et al. [15] incorporated MATLAB and CoNum software in Differential Equation course. The course was conducted in two different modes, either at the university campus or at students' industrial working place. Students attended classes at the university or sometimes the lecturers traveled to the industrial centre, or classes were conducted using video conference. CoNum is an educational software of numerical computation which was developed at the University of Minho and was applied due to its simplicity and user-friendly interfaced programmed from C++ to windows environment. MATLAB, on the other hand, is well known in dealing with matrix operations as well as to solve a wide variety of problems. CoNum was used to explore the Runge-Kutta methods and the results were presented in MATLAB. Three assignments represented 40% of the grade course whereas a final project represented 60% of the total course which were solved using the software. Students were exposed to the ability to use numerical packages and to solve real problems with effective tools. The positive experiences attracted students to continue to work on their tasks.

Shacham et al. [16] used an exercise involving numerical solutions for a system of Ordinary Differential Equations to find out the difficulties of the algorithm to solve a problem and to identify appropriate parameters for the algorithm. This exercise was used to exemplify that computer solutions can be incorrect or inaccurate and thus it is necessary to validate it. The Runge-Kutta algorithm for an "interacting tanks" simulation problem was analyzed using mathematical software packages Polymath 6.1 and MATLAB. Firstly, the algorithm was tested in Polymath 6.1 and then the same algorithm was tested in MATLAB to obtain the same result. Secondly, the appropriateness of the default parameter was investigated. Later, the stiffness of the problem was tested and lastly the solution with stiff algorithms and non-stiff algorithms was compared. This exercise was aimed to illustrate that both analysis and validation of the results are the essence in problem solving and trusting the software package.

Abichandani et al. [17] exemplified the potential of the scientific software MATLAB in laboratory exercise conducted by second-year engineering students at Drexel University. ENGR 232 Dynamics Engineering Systems course introduces modeling, simulation and analysis of dynamic systems using the concepts from the theory of Ordinary Differential Equations. Two one-hour lectures and a two-hour laboratory session were conducted every week. Modeling problems based on the concept during lectures were given in the laboratory session. Students needed to answer questions based on the system and also verified the results they obtained. They needed to plot trajectories and conduct the sensitivity analysis. One of the laboratory exercises introduced a geosynchronous satellite orbital entry problem. MATLAB helped students to understand the behavior of the physical system easier instead of the traditional methods. It was concluded that scientific software should be included in all stages of the engineering curriculum especially in design projects to provide alternatives and to foster sound analysis.

Maat & Zakaria [18] carried out a study with ten engineering technology students to solve Ordinary Differential Equations questions using traditional methods versus technological tool, particularly Maple. Integrating Maple enhanced students' understanding, increased their interest in Mathematics and developed their creative thinking. Time was saved with Maple to solve tedious and complex calculations. Therefore students had the opportunity to interact with other students as well as their lecturers. Maple was an aid of producing interactive activities while learning the lesson. Students were able to link mathematical understanding with real-life engineering applications. Although students were lacking in basic Calculus, yet lecturer managed to build their confidence in solving Ordinary Differential Equations problems.

Zeynivannezad[19] conducted teaching experiments to enhance the mathematical thinking powers in learning Differential Equations through a computer algebra system. A test was conducted prior to the teaching experiments. Questions were designed to identify procedural parts, modeling and interpretation and solutions of the Differential Equations. Findings showed that almost all the students did not answer the questions based on graphical solutions. Maxima software was used to conduct teaching experiments especially in drawing graphs. The procedures of drawing graphs and the visualization of the graph enhanced students' conceptual understanding of Differential Equations.

V. Application of Mathematica in Engineering Mathematics

Meagher [20] investigated the processes of learning in a computer algebra system environment for college students learning Calculus through a case study involving three students. The Rotman Model of Mathematical Reasoning was applied as the macro-framework to view the place of technology in the computer algebra system classroom and to view the journey of each student across the quarter. The Pirie-Kieren Model for the Growth of Mathematical Understanding was applied as the micro-framework to examine specific learning episodes as they occurred in the classroom. Students who took their first Calculus class had the option either to attend the traditional lecture class or choose to take a computer-based class called Calculus & Mathematica. The Calculus & Mathematica class of ten weeks per term used materials developed at The Ohio State University and The University of Illinois (Urbana-Champaign). Students from the Calculus and

Mathematica course took up self-directed learning by using a CD as an interactive textbook. Lecturers and teaching assistants in the laboratory only responded to students' questions rather than to the lecture. One module was given for a week, whereby students worked in a group of three. Four sections were allocated in each module: (i) Basics; (ii) Tutorial; (iii) Give it a Try; and (iv) Literacy. Students worked individually for the first two sections to learn and practice the basic concepts while in the other two sections the students worked in a group of three. The third section was done with the help of Mathematica where students submitted their work to get grades. For the last section, the submission for grading was also required but without the aid of Mathematica. The group's discussions' audio tape and video capture of computer screens were the primary source in this study. Observations, interviews, grounded survey document analyses were undertaken in this qualitative research. Although some students were very positive about the computer algebra system, many who were familiar with the graphic calculator claimed that the Computer algebra system was too much for calculation. Case study revealed that Mathematica was seen to be a burden in calculation and a student could not concentrate on the concept of Calculus. Although two other students agreed with the first part, they did see "true" Mathematica as residing. Students developed resistance because there was no direct training given in using the software. Some students expressed considerable frustration to learn Mathematica and Calculus. However, students did have to take some time to get used to Mathematica and eventually understand that Mathematica was more powerful in calculation compared to graphics calculator.

Adair & Jaeger [21] integrated the computer algebra system Mathematica into Engineering Mathematics modules especially Vector Calculus and Partially Differential Equations. Although real engineering examples were taught in the Engineering curriculum, the boundary between the two still existed. By incorporating Mathematica into the Engineering Mathematics module, students have a better understanding on how engineers face real-life scenario and thus increase their skills and interest towards engineering. Following a pre-test, 136 students were divided into the control group and experimental group. The experimental group conducted six laboratories in a twelve-week period while the control group had extra tutorials. At the end of the course the students performed a post-test. The questions for pre and posttests were the same except for the fact that there were extra two subjective questions in the post-test. Mathematica made no difference to the understanding of 'simple' concepts of Engineering Mathematics but it did make a significant difference in the understanding of 'difficult' concepts in Engineering Mathematics.

VI. Conclusions

There are many applications of Integrated computer software in the teaching and learning of engineering mathematics. Types of software used depend on the subject learning. For Vector Calculus, more researchers applied MAPLE in their teaching as compared to MATLAB. However for both Linear Algebra and Ordinary Differential Equation, MATLAB is equally applied. Mathematica, on the other hand, is also suitable for all the engineering mathematics subjects. Other researchers combined both of the existing softwares with their own customized software. In some of the applications, problems were reported on the attitude level of students towards technology and the time taken to acquire the knowledge and skills. For simple mathematical problems and calculations, the usage of the software is not significant. However, most of the students found that these software can be helpful in the understanding of complex concept and solving tedious calculation.

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